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THERMALLY DEVELOPING LAMINAR FLOW INSIDE RECTANGULAR DUCTS SUBMITTED TO BOUNDARY CONDITIONS OF THIRD KIND

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Abstract. *Thermally developing, hydrodynamically developed, laminar forced convection inside rectangular ducts, subject to boundary conditions of third kind, is analytically studied by using the generalized integral transform technique, allowing for the solution of a convection-diffusion problem with non-separable eigenvalue problem. Constant fluid properties, high Peclet number and negligible viscous dissipation assumptions are utilized. Transforming the energy equation for the unknown temperature profile by the use of the integral transform technique results a coupled system of first order ordinary differential equations for the unknown transformed-temperature distribution. That system is then solved and the temperature profile can be obtained by using the inversion formula. Reference results were established for quantities of practical interest within thermal entry region, for a wide range of axial variable, various aspect ratios and Biot numbers. Among that, thermal quantities are bulk fluid temperature, average wall temperature, average and local Nusselt numbers and thermal entry length. The accuracy of previously reported results was then critically examined, for both the developing and fully developed regions.*

Keywords: *Rectangular channel, Integral transform technique, Forced convection.*

1. INTRODUCTION

Heat transfer solutions for laminar forced flow inside ducts of various shapes is of great interest to the design of compact heat exchangers and several other low Reynolds number flow heat exchange devices. In Shah & London (1978) there are many references for laminar forced convection heat transfer in the thermal entry region of rectangular channel submitted to boundary conditions of first and second kind. A more realistic condition in many applications would be the boundary conditions of third kind, and the establishment of benchmark results through a analytical solutions is quite desirable for both reference purposes and validation of direct numerical schemes. Özisik & Murray (1974) did an extension to the classical integral transform technique (CITT), introducing the so-called generalized integral transform technique (GITT) to solve diffusion problems. For more detailed explanation about the CITT and GITT we recommend Mikhailov & Özisik (1984) and Cotta (1993), respectively.

Three-dimensional convection-diffusion problems within irregularly shaped geometries were solved by Aparecido & Cotta (1990a, 1992). Also, Aparecido & Cotta (1990b) applied that technique to solve laminar forced convection within rectangular shaped cross-section tubes for boundary condition of the first kind.The present contribution attempts to give more results of this problem by extending the ideas in the so-called generalized integral transform technique applied to laminar flow inside rectangular ducts submitted to boundary condition of third kind.

An analysis of convergence was done and a set of benchmark results established for thermal quantities, within a wide range of dimensionless axial coordinate. Previous reported results from direct numerical approaches are then critically examined.

2. ANALYSIS

We consider laminar flow of a Newtonian fluid inside a rectangular channel of sides 2a and 2b, with a fully developed velocity profile and subjected to boundary conditions of third kind. For thermally developing flow the associated energy equation is written in dimensionless form as

$$U(X,Y)\frac{\partial\theta(X,Y,Z)}{\partial Z}=\frac{\partial^2\theta}{\partial X^2}+\frac{\partial^2\theta}{\partial Y^2},\quad 0<X<\alpha,\quad 0<Y<\beta,\quad Z>0 \tag{1}$$

with inlet and boundary conditions given, respectively, by

$$\theta(X,Y,0)=1,\quad 0\le X\le\alpha,\quad 0\le Y\le\beta \tag{2a}$$

$$\left.\frac{\partial\theta(X,Y,Z)}{\partial X}\right|_{X=0}=0,\qquad \left.\frac{\partial\theta(X,Y,Z)}{\partial X}\right|_{X=\alpha}+Bi\theta(0,Y,Z)=0,\qquad Z>0; \tag{2b,c}$$

$$\left.\frac{\partial\theta(X,Y,Z)}{\partial Y}\right|_{Y=0}=0,\qquad \left.\frac{\partial\theta(X,Y,Z)}{\partial Y}\right|_{Y=\alpha}+Bi\theta(X,0,Z)=0,\qquad Z>0. \tag{2d,e}$$

where the following dimensionless groups were defined

$$\theta(X,Y,Z)=\frac{T(x,y,z)-T_\infty}{T_I-T_\infty},\quad X=\frac{x}{D_h},\qquad Y=\frac{y}{D_h},\qquad U(X,Y)=\frac{u(x,y)}{u_m},$$
$$Z=\frac{z}{D_hPe},\qquad Pe=\frac{\rho c_p}{k}u_mD_h,\ \alpha=\frac{a}{D_h},\qquad \beta=\frac{b}{D_h},\qquad Bi=\frac{hD_h}{k}. \tag{3}$$

The dimensionless velocity profile is given as an infinite series (Shah & London, 1978)

$$U(X,Y)=A^*(\alpha^*)\sum_{k=1,3,\ldots}^{\infty}B_kF_k(Y)G_k(X) \tag{4a}$$

where $\alpha^*=2b/2a=$ aspect ratio , $a_k=k\pi/2\alpha$ (4b,c) and, $$A^*(\alpha^*) = \frac{48}{\pi^3\left[1 - \dfrac{192}{\pi^5\alpha^*}\displaystyle\sum_{k=1,3,\ldots}^{\infty}\dfrac{\tanh(a_k\beta)}{k^5}\right]},\tag{4d}$$

$$B_k = \frac{(-1)^{(k-1)/2}}{k^3}, \qquad G_k(X) = \cos(a_k X), \qquad F_k = 1 - \frac{\cosh(a_k Y)}{\cosh(a_k\beta)} \tag{4e,f,g}$$

The exact solution of problem (1) through well-known analytical methods, such as the classical integral transform technique, is not possible due to the non-separable nature of the velocity profile and consequently, of the related eigenvalue problem. However, the advanced ideas on the generalized integral transform technique can be modified to allow for an analytical treatment of the present problem as now demonstrated (Mikhailov & Ozisik, 1984 and Cotta, 1993). First, the difficulties associated with the eigenvalue problem are alleviated by choosing the following auxiliary problems (Aparecido, 1997)

$$\frac{d^2\psi(\mu,X)}{d^2X} + \mu^2\psi(\mu,X) = 0, \qquad 0 < X < \alpha; \tag{5a}$$

$$\left.\frac{d\psi(\mu,X)}{dX}\right|_{X=0} = 0, \qquad \left.\frac{d\psi(\mu,X)}{dX}\right|_{X=\alpha} + Bi\,\psi(\mu,\alpha) = 0\ ; \tag{5b,c}$$

and, $$\frac{d^2\phi(\lambda,Y)}{d^2Y} + \lambda^2\phi(\lambda,Y) = 0, \qquad 0 < Y < \beta; \tag{6a}$$

$$\left.\frac{d\phi(\lambda,Y)}{dY}\right|_{Y=0} = 0, \qquad \left.\frac{d\phi(\lambda,Y)}{dY}\right|_{Y=\beta} + Bi\,\phi(\lambda,\beta) = 0; \tag{6b,c}$$

which are readily solved to yield eigenfunctions, normalization constants, and eigenconditions

$$\psi_i(X) = C_i\cos(\mu_i X), \qquad C_i = \left[\frac{2(\mu_i^2 + Bi^2)}{\alpha(\mu_i^2 + Bi^2) + Bi}\right]^{1/2}, \qquad \mu_i\tan(\mu_i\alpha) = Bi; \tag{7a,b,c}$$

$$\phi_i(Y) = D_m\cos(\lambda_m Y), \qquad D_m = \left[\frac{2(\lambda_m^2 + Bi^2)}{\beta(\lambda_m^2 + Bi^2) + Bi}\right]^{1/2}, \qquad \lambda_m\tan(\lambda_m\beta) = Bi. \tag{8a,b,c}$$

Problems represented by Eqs. (5) and Eqs. (6) above allow the establishment of the following integral transform pair.

Transform: $$\overline{\overline{\theta}}_{im}(Z) = \int_0^\alpha\int_0^\beta \psi_i(X)\phi_m(Y)\theta(X,Y,Z)\,dY\,dX \tag{9}$$

Inversion: $$\theta(X,Y,Z) = \sum_{i=1}^{\infty}\sum_{m=1}^{\infty}\psi_i(X)\phi_m(Y)\overline{\overline{\theta}}_{im}(Z) \tag{10}$$

Multiplying Eq. (1) by $\psi_i(X)\phi_m(Y)$ employing the inversion formula, Eq. (10), and integrating over whole domain it becomes

$$\sum_{j=1}^{\infty}\sum_{n=1}^{\infty} D_{ijmn}\frac{d\overline{\tilde{\theta}}_{jn}(Z)}{dZ}+(\mu_i^2+\lambda_m^2)\overline{\tilde{\theta}}_{im}(Z)=0\,,\qquad Z>0\,,\ i,m=1,2,... \tag{11}$$

where

$$D_{ijmn}=\int_0^{\alpha}\int_0^{\beta}\psi_i(X)\psi_j(X)\phi_m(Y)\phi_n(Y)U(X,Y)dYdX \tag{12}$$

while the transform of inlet conditions becomes

$$\overline{\tilde{\theta}}_{im}(0)=C_i D_m\frac{sin(\mu_i\alpha)sin(\lambda_m\beta)}{\mu_i\lambda_m}=\overline{\tilde{g}}_{im}\,. \tag{13}$$

The double integral in Eq. (12) is done to give

$$D_{ijmn}=A^*(\alpha^*)\sum_{k=1,3,...}^{\infty}B_k\psi_{ijk}\psi^*_{mnk} \tag{14}$$

where

$$\psi_{ijk}=\frac{C_iC_j}{4}\left\{\frac{sin[(a_k+\mu_i+\mu_j)\alpha]}{a_k+\mu_i+\mu_j}+\frac{sin[(a_k+\mu_i-\mu_j)\alpha]}{a_k+\mu_i-\mu_j}+\right.$$
$$\left.+\frac{sin[(a_k-\mu_i+\mu_j)\alpha]}{a_k-\mu_i+\mu_j}+\frac{sin[(-a_k+\mu_i+\mu_j)\alpha]}{-a_k+\mu_i+\mu_j}\right\} \tag{15}$$

and

$$\psi^*_{mnk}=\delta_{mn}+\frac{D_mD_n}{2}\left\{\frac{a_k\tanh(a_k\beta)\cos[(\lambda_m-\lambda_n)\beta]+(\lambda_m-\lambda_n)sin[(\lambda_m-\lambda_n)\beta]}{a_k^2+(\lambda_m-\lambda_n)^2}+\right.$$
$$\left.+\frac{a_k\tanh(a_k\beta)\cos[(\lambda_m+\lambda_n)\beta]+(\lambda_m+\lambda_n)sin[(\lambda_m+\lambda_n)\beta]}{a_k^2+(\lambda_m+\lambda_n)^2}\right\} \tag{16}$$

Equation (11) above provides a denumerable system of coupled ordinary differential equations with constant coefficients, to be solved for obtaining the transformed potentials. For sake of obtaining numerical results, the infinite system has to be truncated to a sufficiently large finite order for the desired convergence. Then, following Aparecido & Cotta (1990), the truncated system is given by

$$\sum_{j=1}^{N}\sum_{n=1}^{N} D_{ijmn}\frac{d\overline{\tilde{\theta}}_{jn}(Z)}{dZ}+(\mu_i^2+\lambda_m^2)\overline{\tilde{\theta}}_{im}(Z)=0\qquad i,m=1,2,...,N, \tag{17}$$

with $\overline{\overline{\tilde{\theta}}}_{im}(0) = \overline{\overline{\tilde{g}}}_{im}$, and the finite system of N^2 coupled equations is given in matrix form

$$\mathbf{P}\mathbf{y}' + \mathbf{E}\mathbf{y} = 0 \text{ with } \mathbf{y}(0) = \mathbf{g} \tag{18}$$

where **P** and **E** are proper representations of the coefficients of Eq. (17), and

$$\mathbf{y} = \{\overline{\overline{\tilde{\theta}}}_{11}(Z), \overline{\overline{\tilde{\theta}}}_{12}(Z), ..., \overline{\overline{\tilde{\theta}}}_{1N}(Z), \overline{\overline{\tilde{\theta}}}_{21}(Z), ..., \overline{\overline{\tilde{\theta}}}_{2N}(Z), ..., \overline{\overline{\tilde{\theta}}}_{N1}(Z), ..., \overline{\overline{\tilde{\theta}}}_{NN}(Z)\}^T \tag{19}$$

$$\mathbf{g} = \{\overline{\overline{\tilde{g}}}_{11}, \overline{\overline{\tilde{g}}}_{12}, ..., \overline{\overline{\tilde{g}}}_{1N}, \overline{\overline{\tilde{g}}}_{21}, ..., \overline{\overline{\tilde{g}}}_{2N}, ..., \overline{\overline{\tilde{g}}}_{N1}, ..., \overline{\overline{\tilde{g}}}_{NN}\}^T \tag{20}$$

Multiplying Eq. (18) by the inverse of matrix **P** that system can then be rewritten in normal form to yield

$$\mathbf{y}' + \mathbf{F}\mathbf{y} = 0 \text{ with } \mathbf{y}(0) = \mathbf{g} \text{ and } \mathbf{F} = \mathbf{P}^{-1}\mathbf{E}. \tag{21}$$

This finite (N^2) system with a constant coefficients matrix, $\mathbf{F}$, was solved by efficient numerical algorithms for initial value problem, such as in subroutine DIVPAG from the IMSL package (Visual Numerics, 1994), with high accuracy.

Once the transformed potentials have been obtained, the inversion formula is recalled to provide the complete temperature profile.

The dimensionless average temperature and average wall temperature are then computed from their definitions

$$\theta_{av}(Z) = \frac{1}{A_c}\int_{A_c} U(X,Y)\theta(X,Y,Z)dA \tag{22}$$

$$\theta_{w,av}(Z) = \frac{1}{p}\int_{p} \theta(X,Y,Z)dp \tag{23}$$

where A_c is the cross-sectional area and p is the perimeter. Performing the above integrals we have

$$\theta_{av}(Z) = \frac{A^*(\alpha^*)}{\alpha\beta}\sum_{i=1}^{N}\sum_{m=1}^{N} Q_{im}\overline{\overline{\tilde{\theta}}}_{im}(Z) \tag{24}$$

$$\theta_{w,av}(Z) = \frac{1}{\alpha+\beta}\sum_{i=1}^{N}\sum_{m=1}^{N} R_{im}\overline{\overline{\tilde{\theta}}}_{im}(Z) \tag{25}$$

where

$$Q_{im} = \sum_{k=1}^{\infty} B_k \frac{C_i D_m}{2}\left[\frac{sin[(\mu_i + a_k)\alpha]}{(\mu_i + a_k)} + \frac{sin[(\mu_i - a_k)\alpha]}{(\mu_i - a_k)}\right]\times \\ \times\left[\frac{sin(\lambda_m\beta)}{\lambda_m} - \frac{a_k \tanh(a_k\beta)\cos(\lambda_m\beta) + \lambda_m sin(\lambda_m\beta)}{a_k^2 + \lambda_m^2}\right] \tag{26}$$

$$R_{im} = C_i D_m \left[\frac{\cos(\mu_i \alpha) sin(\lambda_m \beta)}{\lambda_m} + \frac{sin(\mu_i \alpha)\cos(\lambda_m \beta)}{\mu_i} \right]. \quad (27)$$

The local Nusselt number can be evaluated by making use of the temperature gradients at the wall integrated over the perimeter, or utilizing the axial gradient of the average temperature, providing the following couple of working formulae

$$Nu_1(Z) = \frac{h_1(z) D_h}{k} = \\ = -\frac{1}{(\alpha+\beta)(\theta_{av} - \theta_{w,av})} \left[\int_0^\alpha \frac{\partial \theta(X,Y,Z)}{\partial Y}\bigg|_{Y=\beta} dX + \int_0^\beta \frac{\partial \theta(X,Y,Z)}{\partial X}\bigg|_{X=\alpha} dY \right] \quad (28)$$

$$Nu_2(Z) = \frac{h_2(z) D_h}{k} = -\frac{1}{4(\theta_{av} - \theta_{w,av})} \frac{d\theta_{av}(Z)}{dZ} \quad (29)$$

or after doing above integrals

$$Nu_1(Z) = \frac{1}{(\alpha+\beta)(\theta_{av} - \theta_{w,av})} \sum_{i=1}^{N} \sum_{m=1}^{N} C_i D_m sin(\mu_i \alpha) sin(\lambda_m \beta) \left[\frac{\lambda_m}{\mu_i} + \frac{\mu_i}{\lambda_m} \right] \overline{\tilde{\theta}}_{im}(Z) \quad (30)$$

$$Nu_2(Z) = -\frac{A^*(\alpha^*)}{4\alpha\beta(\theta_{av} - \theta_{w,av})} \sum_{i=1}^{N} \sum_{m=1}^{N} Q_{im} \frac{d\overline{\tilde{\theta}}_{im}(Z)}{dZ} \quad (31)$$

The average Nusselt numbers are then computed from

$$Nu_{av,1}(Z) = \frac{1}{Z}\int_0^Z Nu_1(Z) dZ, \qquad Nu_{av,2}(Z) = \frac{1}{Z}\int_0^Z Nu_2(Z) dZ. \quad (32a,b)$$

From here, where appears Nusselt number $Nu(Z)$ it refers to $Nu_2(Z)$.

3. RESULTS AND DISCUSSION

System (21) was solved for $N \leq 20$ to illustrate the convergence behavior of the present approach, within a wide range of Z, from 10^{-4} to 10^0. Figure 1 shows $Nu(Z)$ for $N = 5, 10, 15, 20$, for a square duct ($\alpha^* = 1$). The Nusselt number computed for $N = 20$ and $N = 15$ are practically coincident for $Z \geq 10^{-4}$, while for $N = 10$, they are coincident for $Z \geq 3\times10^{-4}$, and for $N = 5$, they are coincident for $Z \geq 2\times10^{-3}$. In the other figures and tables the results have been computed for $N = 15$.

Table 1 presents a comparison of limiting Nusselt numbers from various sources, compiled in Shah & London (1978), Aparecido & Cotta (1990) and from Javeri (1978). The results of Miles & Shih for a 40x40 finite difference grid appear to be more accurate than those by Schmidt. The present solution is in a good agreement with the results. Table 2 presents a comparison of limiting Nusselt numbers from Javeri (1978) for $Bi^* = 1$, where $Bi^* = ah/k$.

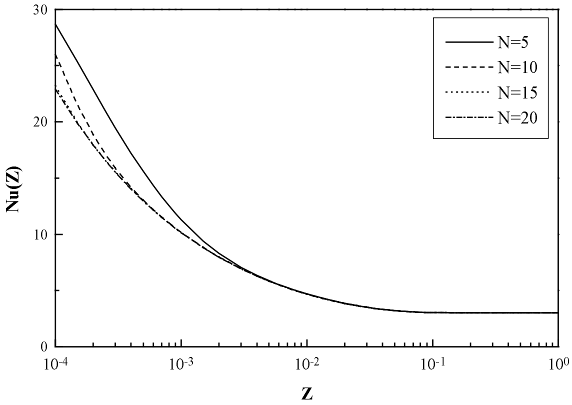

Figure 1: Convergence of local Nusselt number for a square duct ($\alpha^* = 1$) and $Bi = 2$.

Table 1: Comparison of limiting Nusselt numbers from different sources and for various aspect ratios, for $Bi = \infty$.

Aspect Ratio α^*	Shah&London, 1978*	Aparecido &Cotta, 1990	Miles&Shih, in Shah&London, 1978	Clark & Kays, 1953	Javeri,1978	Present solution
1	2.979	2.978	2.976	2.89	2.981	2.978
1/2	3.389	3.392	3.391	3.39	3.393	3.392
1/4	4.435	4.440	4.439	-	4.475	4.439
1/8	5.596	5.607	5.597	-	5.684	5.599

* $Nu(\infty) = 7.541(1 - 2.610\alpha^* + 4.970\alpha^{*2} - 5.119\alpha^{*3} + 2.702\alpha^{*4} - 0.548\alpha^{*5})$

Table 2: Comparison of limiting Nusselt numbers from Javeri (1978) for various aspect ratios.

Aspect Ratio	Javeri,1978	Present solution
1	3.014	3.018
1/2	3.141	3.142
1/4	3.845	3.860
1/8	5.131	5.106

* For $Bi^* = 1$, where $Bi^* = ah/k$.

Table 3 presents results for thermal entry length and, Table 4 presents local and average Nusselt number, and dimensionless bulk fluid and average wall temperature for various Biot numbers, for $\alpha^* = 1$.

Table 3: Results for thermal entry length (L_{th}) for various Biot numbers, for $\alpha^* = 1$.

Bi	∞	200	20	2	0.2	0.02
L_{th}	0.043	0.044	0.047	0.054	0.056	0.057

Figures 2 correspond to the dimensionless average temperature profiles for rectangular ducts with different aspect ratios, for $Bi = 2$. Figures 3(a) and 3(b) show the local and average Nusselt number, respectively, in the thermal entry region of rectangular ducts with different aspect ratios, for $Bi = 2$.

Table 4: Results for local, average Nusselt number, and dimensionless average, wall average temperature for various Biot numbers, for $\alpha^* = 1$.

Bi	Z	*Nu*(Z)	Nu_{av}(Z)	θ_{av}(Z)	$\theta_{w,av}$(Z)
	0.0001	21.802	30.418	0.989	0.000
	0.0005	11.841	17.999	0.965	0.000
∞	0.001	9.298	14.187	0.945	0.000
	0.01	4.347	6.475	0.772	0.000
	0.1	2.982	3.521	0.245	0.000
	1	2.978	3.030	0.000	0.000
	0.0001	22.300	33.482	0.989	0.104
	0.0005	12.018	18.808	0.968	0.055
200	0.001	9.407	14.660	0.948	0.043
	0.01	4.366	6.556	0.778	0.017
	0.1	2.982	3.533	0.251	0.004
	1	2.977	3.033	0.000	0.000
	0.0001	23.033	38.638	0.995	0.535
	0.0005	12.700	20.515	0.981	0.381
20	0.001	9.886	15.795	0.967	0.320
	0.01	4.488	6.859	0.823	0.151
	0.1	2.984	3.590	0.303	0.039
	1	2.976	3.038	0.000	0.000
	0.0001	23.228	39.033	0.999	0.920
	0.0005	12.976	20.820	0.996	0.863
2	0.001	10.156	16.084	0.993	0.830
	0.01	4.687	7.092	0.941	0.660
	0.1	3.038	3.708	0.601	0.362
	1	3.017	3.087	0.008	0.005
	0.0001	23.199	39.000	1.000	0.991
	0.0005	12.969	20.804	1.000	0.984
0.2	0.001	10.160	16.075	0.999	0.980
	0.01	4.735	7.121	0.992	0.952
	0.1	3.101	3.771	0.927	0.871
	1	3.075	3.146	0.472	0.443
	0.0001	23.210	39.029	1.000	0.999
	0.0005	12.972	20.813	1.000	0.998
0.02	0.001	10.162	16.081	1.000	0.998
	0.01	4.741	7.125	0.999	0.995
	0.1	3.113	3.780	0.992	0.986
	1	3.086	3.156	0.924	0.918

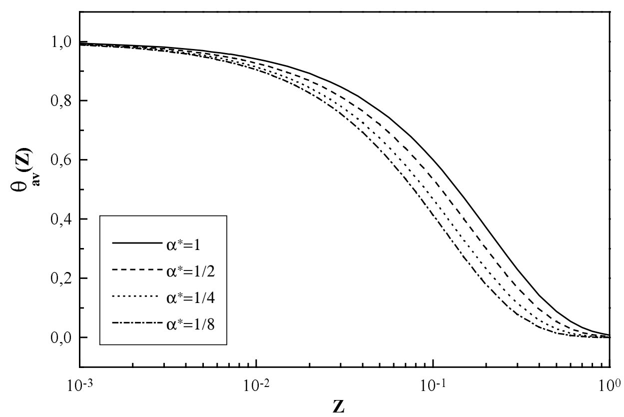

Figure 2: Dimensionless average temperature.

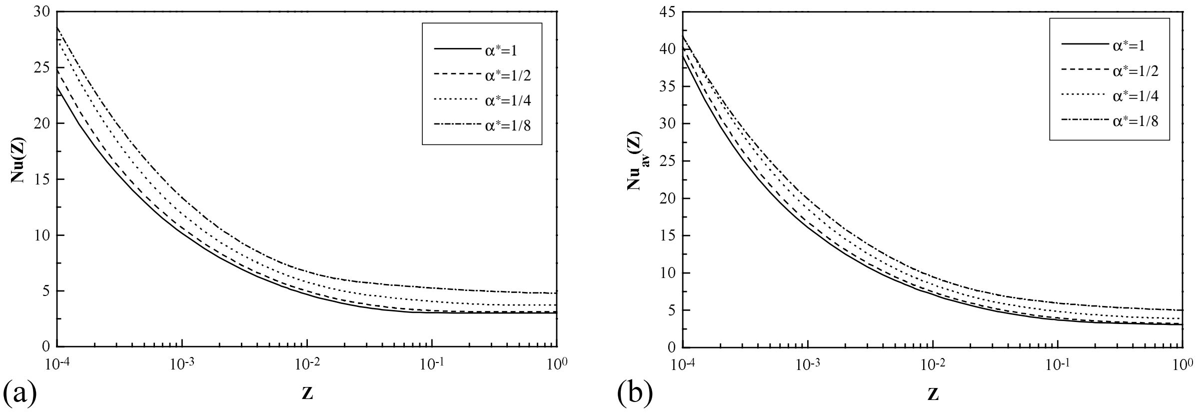

Figure 3: (a) Local and (b) average Nusselt number.

Figure 4(a) presents results for local Nusselt number, and Fig. 4(b) presents results for dimensionless average temperature and average wall temperature profiles in the thermal entry region of square ducts for different Biot numbers.

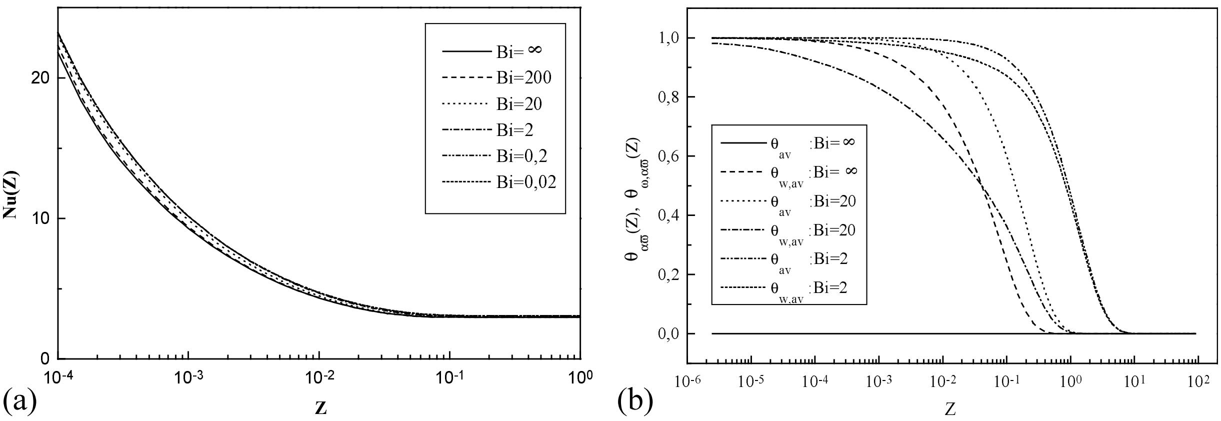

Figure 4: (a) Local Nusselt number and (b) dimensionless average temperature and average wall temperature profiles

Figure 5 presents a comparison of local Nusselt number in the thermal entry region of square ducts, for $Bi = \infty$. Present solution coincides with Chandrupatla & Sastri (1977) and diverges from Javeri (1978), for about $Z < 2.10^{-3}$.

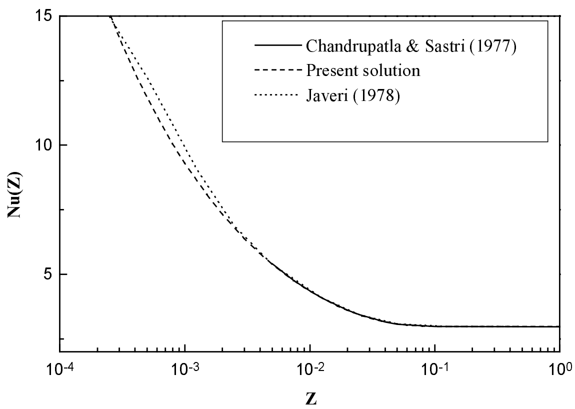

Figure 5: Comparison of local Nusselt number.

The data obtained are in a good agreement with literature, providing a set of benchmark results both for reference purposes and calibration of purely numerical schemes devised for more involved problems. The present approach demonstrated to be relatively cheap, in the range of N considered, running in a personal computer taking less than a minute for each run.

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